Multiscale Conjugate Gradient
SAGEEP11 Rayfract® 3.34 & 4.01

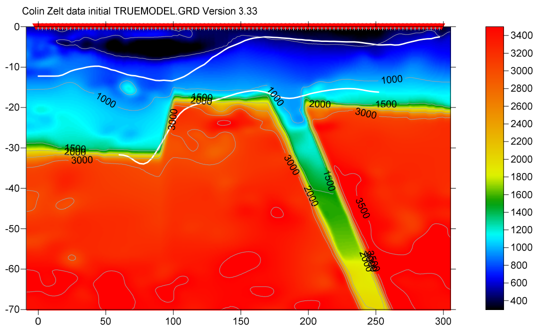

Fig. 1 : Basement step with fault model. See [Zelt et al. 2013 : blind test of refraction methods]().

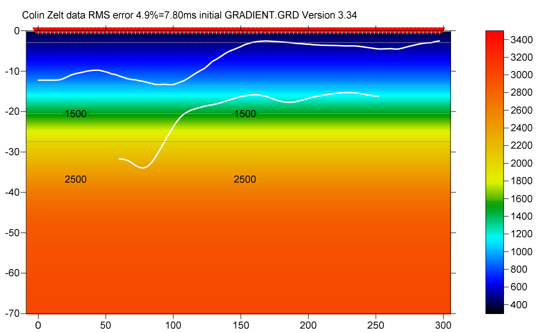

Fig. 2 : 1D-Gradient starting model : laterally average [DeltatV]() inversion. RMS error 4.9%=7.80 ms.

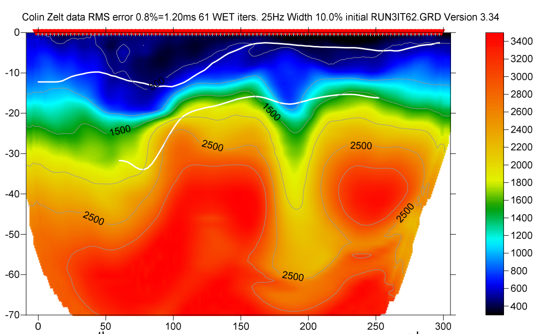

Fig. 3 : 4th WET run. Starting model is 3rd run output. Wavepath width 10%. RMS error 0.8%=1.20ms.

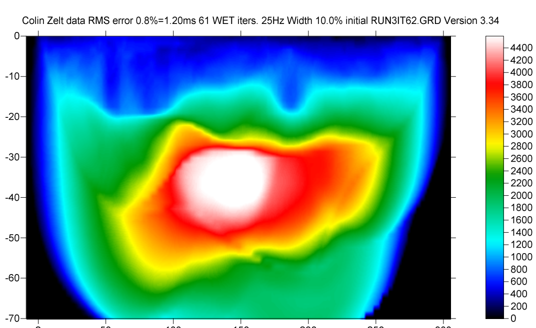

Fig. 4 : WET wavepath coverage plot for Fig. 3. Unit is wavepaths per pixel. Wavepath width 10%@25Hz.

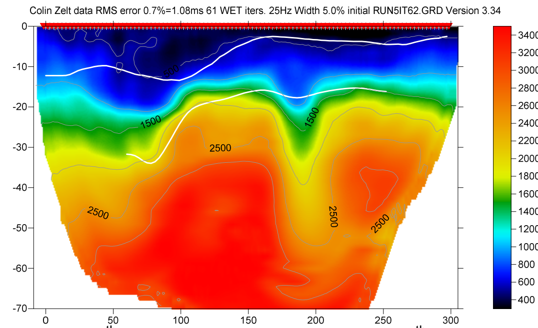

Fig. 5 : 6th WET run. Starting model is 5th run output. Wavepath width 5%. RMS error 0.7%=1.08ms.

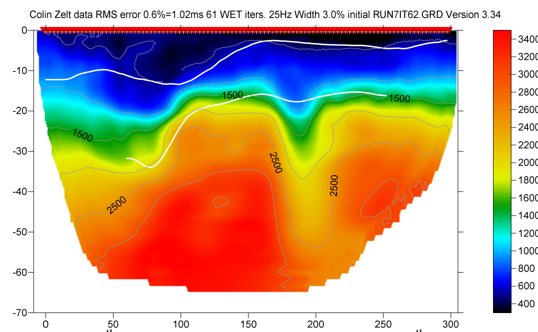

Fig. 6 : 8th WET run. Starting model is 7th run output. Wavepath width 3%. RMS error 0.6%=1.02ms.

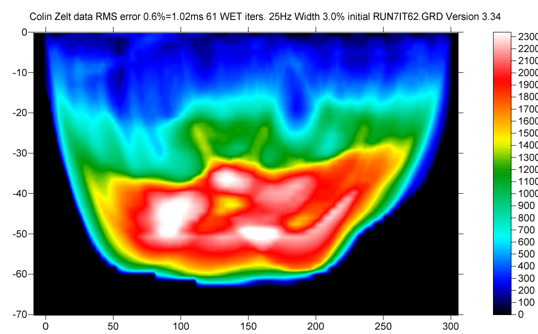

Fig. 7 : WET wavepath coverage plot for Fig. 6. Unit is wavepaths per pixel. Wavepath width 3%@25Hz.

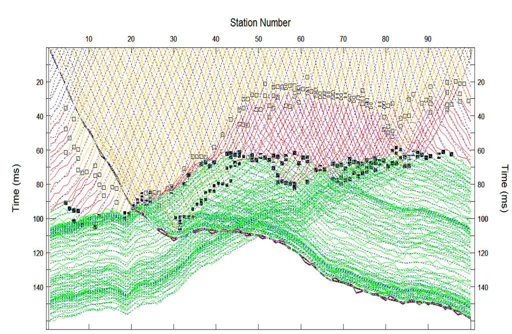

Fig. 8 : True times (colored solid curves forward modeled for Fig. 1 **with added Gaussian noise**) and inverted times (dashed blue curves for Fig. 6)

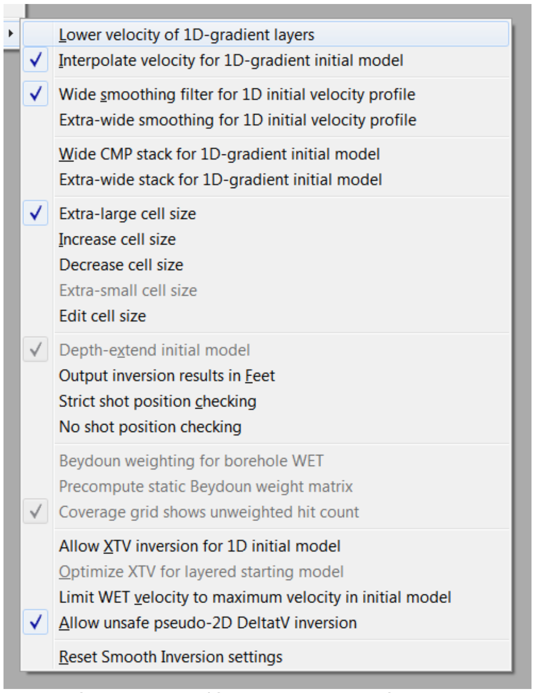

Fig. 9 : Smooth invert|Smooth inversion Settings

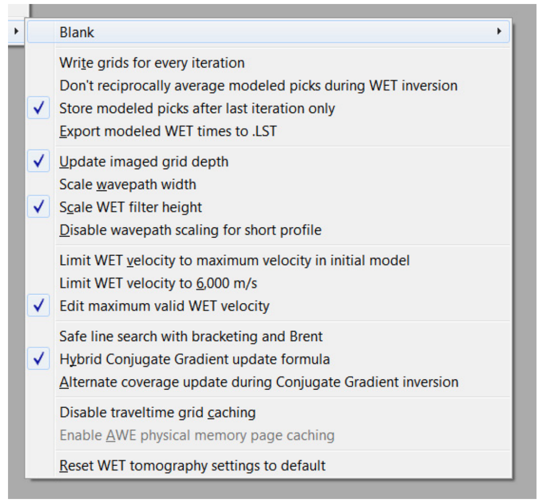

Fig. 10 : WET Tomo|WET tomography Settings

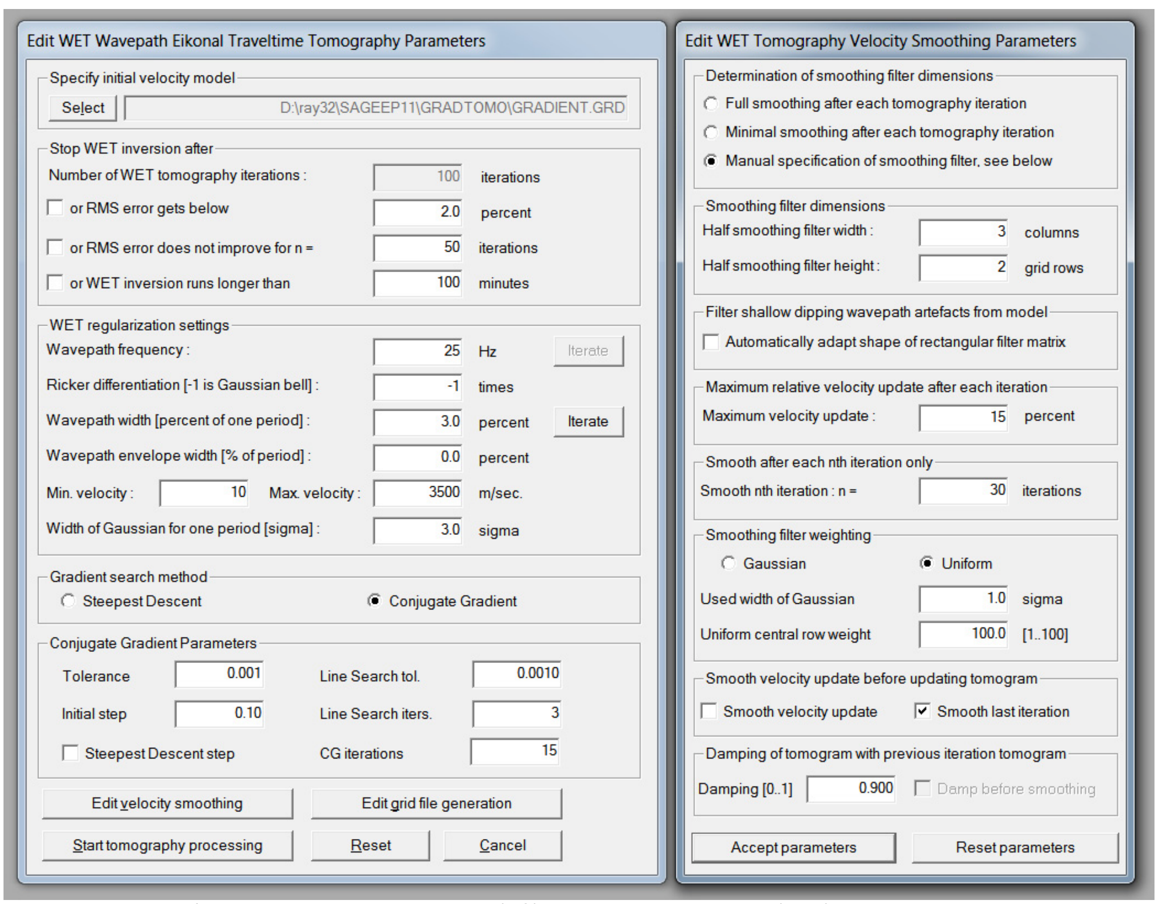

Fig. 11 : WET Tomo|Interactive WET tomography… (left), Edit velocity smoothing (right)

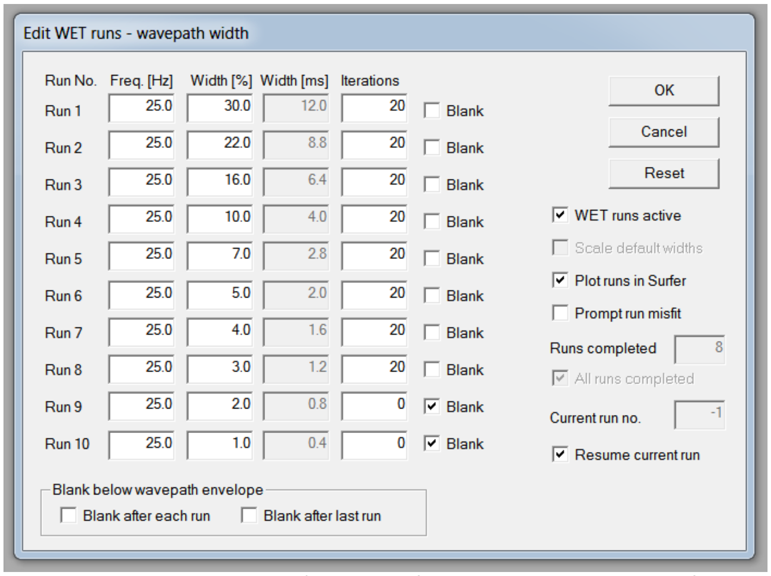

Fig. 12 : multirun WET settings (WET Tomo|Interactive WET tomography|Iterate button)

To obtain Fig. 2 to Fig. 8 proceed as follows :

- Download [334CGWETGrad_smooth30_seis32.zip]() archive and unzip into directory `C:\RAY32\SAGEEP11`
- Select *File|Open Profile…* & database schema `C:\RAY32\SAGEEP11\SEIS32.DBD` . Click *Open button*.
- Check *Smooth invert|Smooth inversion Settings|**Extra-large cell size***
- Setup other settings in *Smooth invert|Smooth inversion Settings* as in Fig. 9
- Uncheck *WET Tomo|WET tomography Settings|**Disable wavepath scaling for short profile***
- Uncheck *WET Tomo|WET tomography Settings|**Scale wavepath width***
- Check *WET Tomo|WET tomography Settings|**Edit maximum valid WET velocity***
- Check *WET Tomo|WET tomography Settings|Store modeled picks after last iteration only*
- Setup other settings in *WET Tomo|WET tomography Settings* as in Fig. 10
- Select *Smooth invert|WET with 1D-gradient initial model* and confirm prompts to obtain 1D-gradient starting model (Fig. 2) and default Smooth inversion output after 20 WET iterations
- Select *WET Tomo|Interactive WET tomography…*
- Click *Select button* and select starting model `C:\RAY32\SAGEEP11\GRADTOMO\GRADIENT.GRD`
- Check *box **Skip every 2nd shot for forward modeling*** and click *button Accept parameters*
- Set *edit field **Wavepath frequency*** to 25Hz and set ***Max. velocity*** to 3,500m/s
- Click *radio button [Conjugate Gradient]()* and uncheck *box **Steepest Descent step***
- Edit other fields in *WET main dialog* as in Fig. 11 (left)
- Click *button **Edit velocity smoothing*** and *radio button **Manual specification of smoothing filter***
- Set ***Half smoothing filter width*** to 3 columns and ***Half smoothing filter height*** to 2 grid rows
- Uncheck box *Automatically adapt shape of rectangular filter matrix*
- Set ***Maximum velocity update*** to 15% and ***Damping*** to 0.9
- Set ***Smooth nth iteration : n =*** to 30 and click *radio button Uniform*
- Set ***Uniform Central row weight*** to 100 and uncheck *box **Smooth velocity update***
- Edit other fields in *Velocity Smoothing Parameters dialog* as in Fig. 11 (right)
- Click *button Accept parameters*
- Click ***Iterate** button* in WET main dialog and click *Reset button* in *Edit WET runs dialog*
- Check ***box WET runs active*** and uncheck *box Blank after last run*
- Edit other fields in *Edit WET runs dialog* as in Fig. 12 and click *OK button*
- Click *Edit grid file generation button* and set *Store each nth iteration only : n* = 50.
- Click *Accept parameters button* and *Start tomography processing button*
- Confirm prompts to obtain output as in Fig. 3 to Fig. 7
- Select *Refractor|Shot breaks* to obtain Fig. 8
- Uncheck *Mapping|Display raytraced traveltimes*

With these velocity smoothing settings (Fig. 11) we obtain robust convergence of *multirun WET inversion* with RMS error quasi-monotonically decreasing from Fig. 2 (4.9%=7.80ms) through Fig. 6 (0.6%=1.02ms). In particular we specify :

- ***Wavepath frequency of 25Hz*** instead of default 50Hz. This makes multirun WET more robust.
- ***Max. velocity = 3,500m/s***. This prevents unrealistic oscillation of Conjugate Gradient WET modeled velocity in basement.
- ***Maximum velocity update = 15%*** instead of default 25%
- ***Damping = 0.9*** equals default 0.9 for *Conjugate Gradient method*
- ***Smooth nth iteration : n = 30*** instead of default 1
- ***Uniform central row weight = 100*** instead of default value 1. This increases the vertical resolution in special case of quasi-horizontal layering in subsurface with ***Uniform*** *Smoothing filter weighting*.
- ***Manual Smoothing filter specification*** with ***Half-width = 3 columns*** and ***Half-height = 2 rows***
- ***No smoothing of velocity update*** before applying the update to the current velocity tomogram

Fig. 3 (wavepath width 10%) is a low-frequency approximation of the true model (Fig. 1) and shows long-wavelength features of the true model. Fig. 6 (wavepath width 3%) is the final high-frequency interpretation of the traveltime data and shows more detail (short wavelength scale). By using above *maximum velocity update* of 15% and *damping* of 0.9 we keep the long wavelength features from Fig. 3 and add shorter wavelength resolution through all WET runs up to Fig. 6. Increasing the *damping* to 0.92 or even 0.95 better suppresses inversion artefacts at bottom of the dipping fault zone.

When using the *Conjugate Gradient search method* try enabling ***WET Tomo|WET tomography Settings|Blank|Blank low coverage after last iteration*** . This will blank out low-velocity edges at the bottom of the tomogram. These can occur because the Conjugate Gradient method combines previous velocity updates from all previous WET iterations (for current run) with the *Steepest Descent direction* for current iteration to determine the update for the current iteration (in new search direction).

To understand the Conjugate Gradient theory regard the velocity update grid as a vector with as many dimensions as there are cells in the tomogram grid i.e.

vector dimensions = column count times row count of grid .

As stated in Zelt et al. 2013 "Uncorrelated Gaussianly-distributed noise with a mean of zero and a standard deviation of 1 ms was added to the synthetic data". This ***added Gaussian noise*** prevents our WET inversion from reaching an even better resolution in the final tomogram Fig. 6.

Runtime for above 8 WET runs 62 WET iterations each = 496 iterations was about ***6 minutes on an Apple iMac*** late 2015 with 2.8 GHz Intel Core i5 processor running Parallels Desktop 11 and Windows 7 Pro. Grid size is 80 rows x 200 columns. X spacing & Y spacing are 1.58m. One grid cell is 1.58m square. This fast runtime enables quasi-interactive variation of *WET smoothing parameters* (Fig. 11).

Try to speedup multirun Conjugate Gradient (CG) *WET inversion* by decreasing ***CG iterations*** from default 15 to 5 or 10 (Fig. 11 left) and/or decreasing ***Line Search iters.*** from default 3 to 2. *CG iterations* is the number of outer loop iterations i_{max}, *Line Search iters.* is the number of inner loop iterations j_{max} in the algorithm shown on page 53 of (Shewchuk, 1994). One inner iteration is done with one *WET iteration*. Our *Tolerance* parameter corresponds to Shewchuk epsilon ε for outer loop termination. Our *Line Search tol.* parameter corresponds to Shewchuk epsilon ϵ used for inner loop termination. Our *Initial step* parameter corresponds to Shewchuk sigma σ used during first *Secant method* iteration in Equation (58) on page 46 of (Shewchuk, 1994).

We have shown that using our default 1D-gradient starting model (Fig. 2) for multiscale tomography gives you a good vertical resolution (Fig. 6) comparable to the true model (Fig. 1). Sheehan et al. 2005 evaluate our 1D-Gradient starting model with synthetic data generated for known subsurface models.

Also our tutorial http://rayfract.com/tutorials/step.pdf written in 2013 using version 3.25 of our software shows that using our default 1D-Gradient starting model with Smooth inversion and 20 or 100 WET iterations gives a good vertical resolution.

Below we show using alternative WET parameters. For Fig. 13 (tutorial sageep11_smooth50.pdf) we used *Steepest Descent* method instead of *Conjugate Gradient* method, *Damping* 0.0 instead of 0.9, *Smooth nth iteration : n* = 50 and above 1D-gradient starting model (Fig. 2). Fig. 14 was obtained with same settings as for Fig. 13 but completely disabling WET smoothing by setting *Smooth nth iteration : n* = 100 and unchecking *Smooth last iteration*. This will normally not work for field recorded data due to measurement errors and too wide shot spacing.

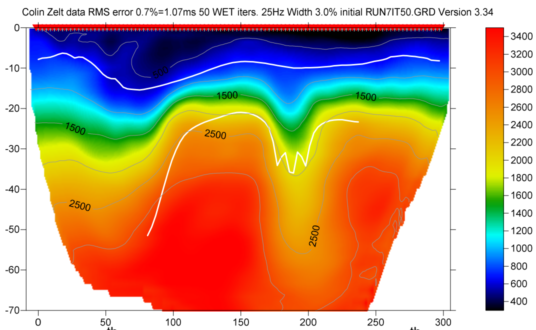

Fig. 13 : 8th WET run. Starting model is 7th run output. Wavepath width 3%. RMS error 0.7%=1.07ms. 1D-gradient starting model. Steepest Descent method. 50 WET iterations per run. Smooth nth iteration : n = 50 . Smooth last iteration unchecked. Damping 0.0 .

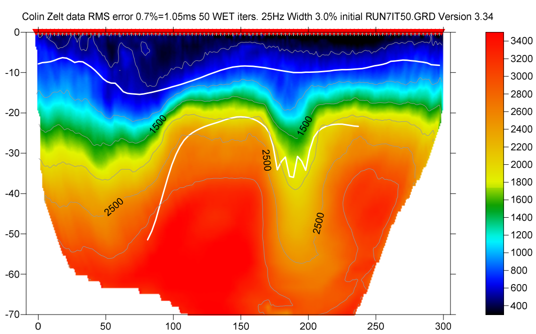

Fig.14 : same as Fig. 13 but Smooth nth iteration : n = 100, no smoothing. RMS error 0.7%=1.05ms .

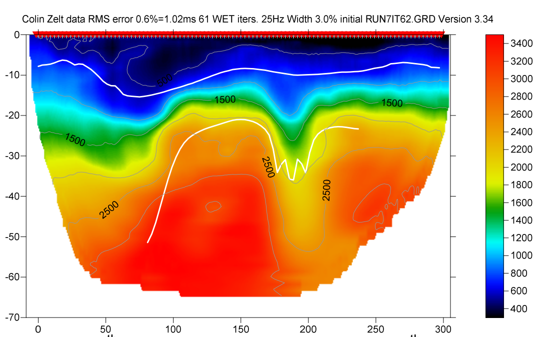

Fig. 15 : 8th WET run. Starting model is 7th run output. Wavepath width 3%. RMS error 0.6%=1.02ms. 1D-gradient starting model. Conjugate Gradient method. Smooth nth iteration : n = 30 . Smooth last iteration unchecked. Damping 0.9 . See Fig. 6.

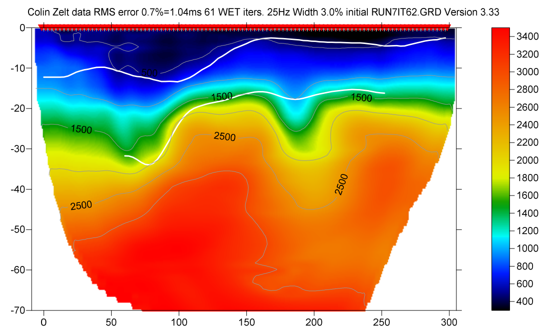

Fig. 16 : same as Fig. 15 but Smooth nth iteration : n = 4 . Smooth last iteration unchecked. Damping 0.9 . RMS error 0.7%=1.04ms. 1D-gradient starting model. Conjugate Gradient method.

For field-recorded data first run our *Smooth inversion* with *Smooth invert|WET with 1D-gradient initial model*, with default 20 and interactive 50 or 100 iterations. Next you can optionally decrease the smoothing and try *Conjugate Gradient* method instead of default *Steepest Descent* method, see above. Try *Minimal smoothing*, *Smooth nth iteration : n* = 4 or 10 and disabling *Smooth velocity update* (Fig. 11).

Quoted from our SAGEEP 2014 paper : "Besides showing the resolution limit of one wavelength (Watanabe et al., 1999), WET blurring also can show the uncertainty caused by bad picks, recording geometry errors, uncorrected trigger delays and out-of-plane refractions. The lower the signal-to-noise ratio, the wider the wavepaths should remain. Ray-based tomography methods tend to overfit the data and often generate artefacts (Zelt et al., 2013)."

Alternatively to the 1D-gradient starting model you can use a ***layered refraction starting model*** with our WET inversion. See tutorial norcal14.pdf .

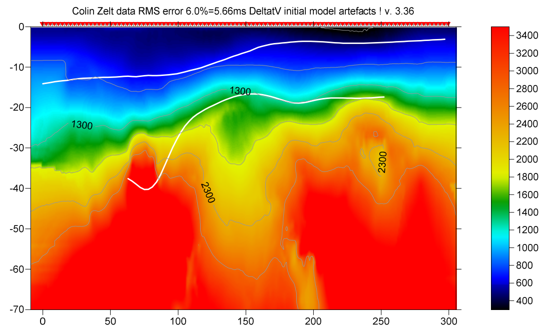

Fig. 16 : DeltatV inversion without WET inversion. CMP stack width 20, Regression over offset stations 5, No weighting of picks in CMP curves. Don't smooth CMP curves. No modified Dix layer inversion.

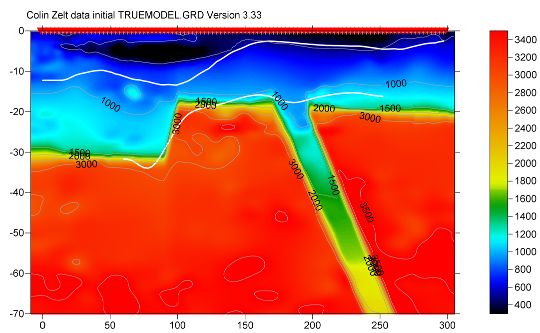

Fig. 17 : Original Basement step with fault model. See [Zelt et al. 2013 : blind test of refraction methods](#)

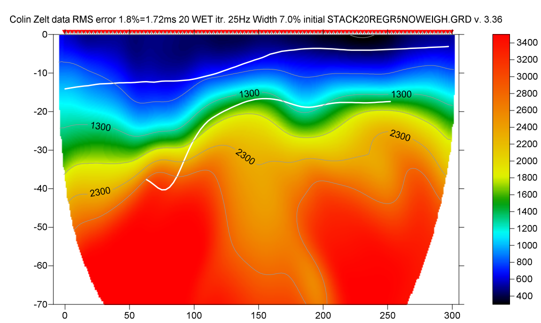

Fig. 18 : 20 WET iterations with starting model Fig. 16. Full WET smoothing Steepest Descent method wavepath frequency 25Hz. Ricker differentiation set to -1 : Gaussian WET update weighting. Wavepath width 7%. Max. WET velocity 3,500 m/s

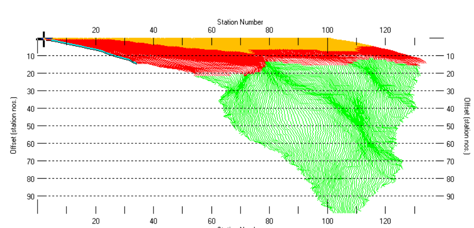

Fig. 19 : Refractor|Midpoint breaks display. Dip of CMP curves zoomed with CTRL+F1. Traces mapped to refractors as in Fig. 20. Station no. range increased to 0..140 with ALT+P.

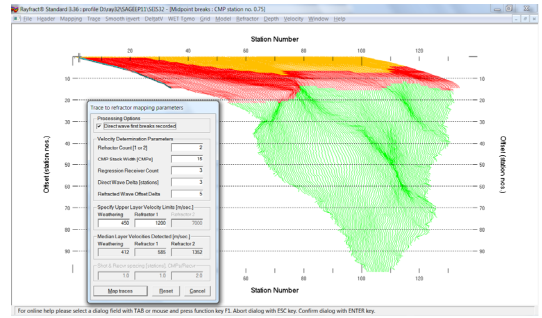

Fig. 20 : map CMP sorted traces to refractors in Refractor|Midpoint breaks with ALT+M. Smooth crossover distance with ALT+G. Note how basement step & dipping fault zone become visible in basement refractor (green).

In Fig. 16 we show the ***pseudo-2D DeltatV starting model*** obtained with *DeltatV|Automatic DeltatV* command.

Note how basement step and dipping fault shown in Fig. 16 are laterally offset from true locations shown in Fig. 17. WET inversion (Fig. 18) helps to improve this offset error.

In Fig. 19&20 we show how to visualize the basement step and dipping fault zone in the raw traveltime data sorted by Common MidPoint CMP before running any inversion.

Steeper dip of CMP traveltime curves means higher apparent velocity. More shallow dip means lower apparent velocity.

Here are the SEIS32.* profile database files for Fig. 16&18. Here are the Surfer .GRD files & .PAR files & Surfer 11 .SRF for Fig. 16&18.

Our modeling in thrust and thrust12 tutorials shows that Smooth inversion and DeltatV inversion can detect dipping fault zones if these zones are wide enough. For an objective comparison of tomographic refraction analysis methods see Zelt et al. 2013 (JEEG, September 2013, Volume 18, Issue 3, pp. 183–194).

We recommend to always use our fail-safe 1D-gradient starting model (Fig. 2 and Fig. 23; Sheehan 2005) to prevent pseudo-2D artefacts in the starting model (Fig. 16).

Alternatively use a ***layered refraction Plus-Minus method starting model*** for our WET inversion. See tutorial Aaknes-1.

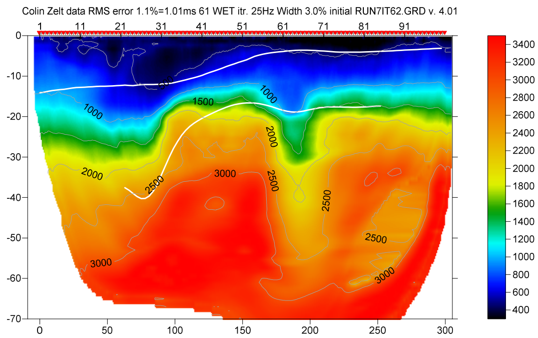

Fig. 21 : 8th WET run. Starting model is 7th run output. Wavepath width 3%. RMS error 1.1%=1.01ms.

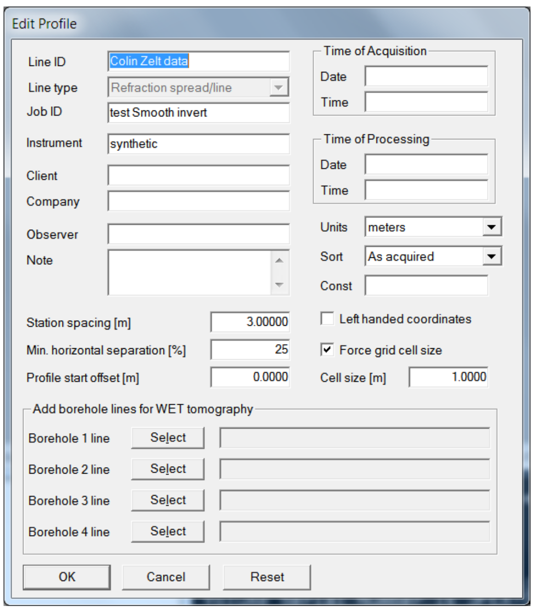

Fig. 22 : force *grid cell size* to 1.0m in *Header|Profile*

We have redone multiscale WET inversion with 1D-gradient starting model (Fig. 21) :

- forced cell size to 1.0m in *Header|Profile* see Fig. 22
- otherwise same DeltatV & Smooth invert & WET settings as in Fig. 11 / as above
- skipped every 2nd shot for WET as above

Compare Fig. 21 obtained with version 4.01 of our software with Fig. 6 obtained with version 3.34. Basement top is better imaged in Fig. 21, due to decreased grid cell size.

Here are the SEIS32.* profile database files for Fig. 21. Here are the Surfer .GRD files & .PAR files & Surfer 11 .SRF for Fig. 21.

Multiscale WET inversion (8 WET runs) with final output shown in Fig. 21 and above settings shown in Fig. 11 etc. with 1D-gradient starting (Fig. 2) model took ***8 minutes on 2017 iMac with Intel Core i5*** @ 2.3 GHz running Windows 7 64-bit Pro in Parallels Desktop 14 using MacOS High Sierra.

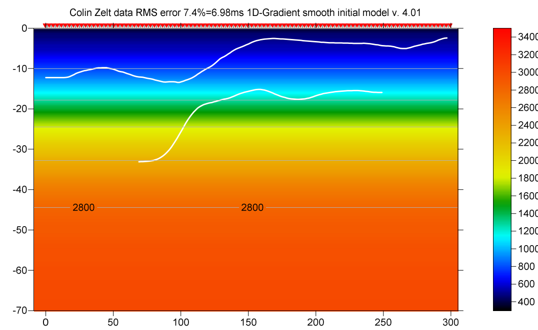

Fig. 23 : 1D-gradient starting model (Sheehan 2005).

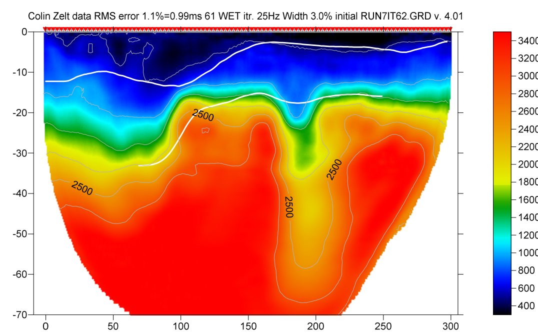

Fig. 24 : Same as Fig. 21 but WDVS engaged at 75Hz (Fig. 27). 8th WET run. Starting model is 7th run. Wavepath width 3%. RMS error 1.1%=0.99 ms.

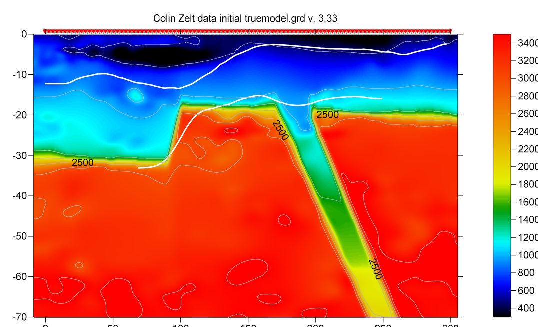

Fig. 25 : True model (Zelt et al. 2013)

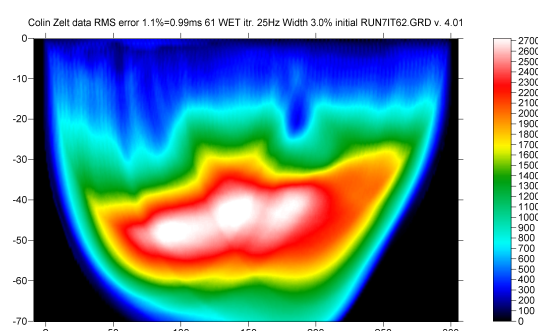

Fig. 26 : WET wavepath coverage plot for Fig. 24.

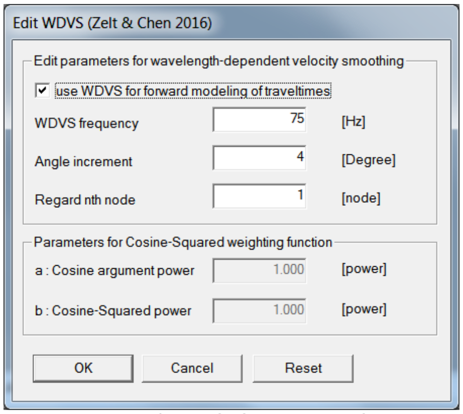

Fig. 27 : Model|WDVS Smoothing. Check Model|Fast WDVS Smoothing. Used for Fig. 24.

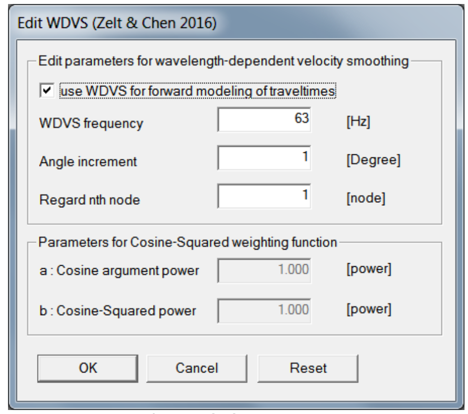

Fig. 31 : Model|WDVS Smoothing. Uncheck Model|Fast WDVS Smoothing. Used for Fig. 32.

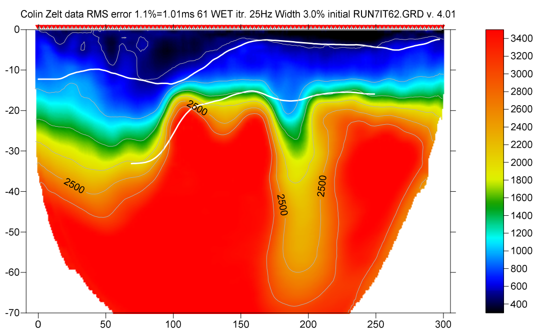

Fig. 28 : same as Fig. 24 but WDVS at 63Hz (Fig. 27). Angle increment=4. Regard nth node=1. Wavepath width 3%. RMS error 1.1%=1.01 ms.

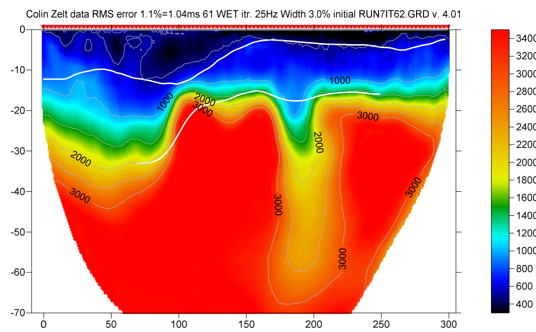

Fig. 32 : same as Fig. 24 but WDVS at 63Hz (Fig. 31) with Model|Fast WDVS Smoothing unchecked. Angle increment=1. Regard nth node=1. Wavepath width 3%. RMS error 1.1%=1.04 ms.

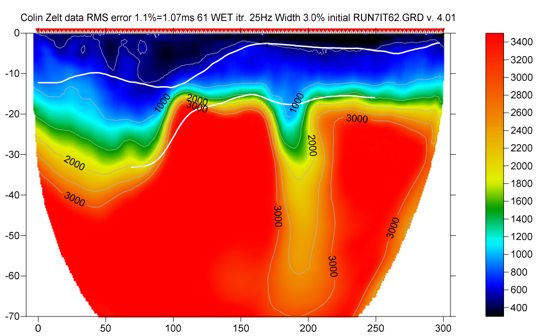

Fig. 29 : same as Fig. 24 but WDVS at 57Hz (Fig. 27). Wavepath width 3%. RMS error 1.1%=1.07ms.

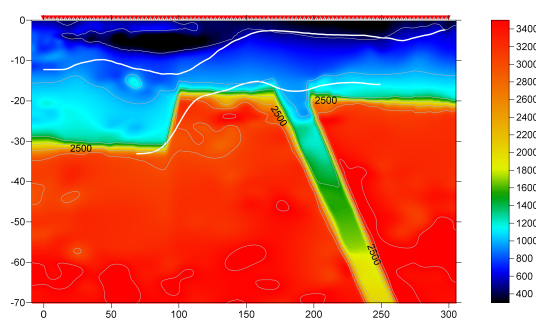

Fig. 33 : True model ([Zelt et al. 2013](#)).

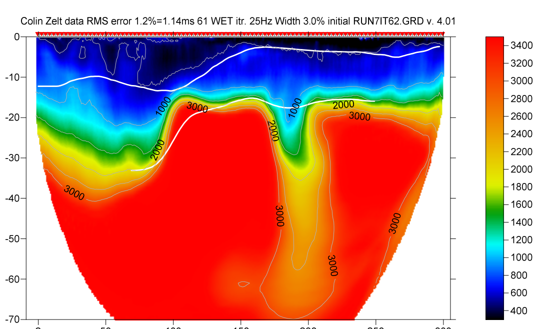

Fig. 30 : same as Fig. 29 but Fast WDVS Smoothing unchecked. Wavepath width 3%. RMS error 1.2%=1.14ms.

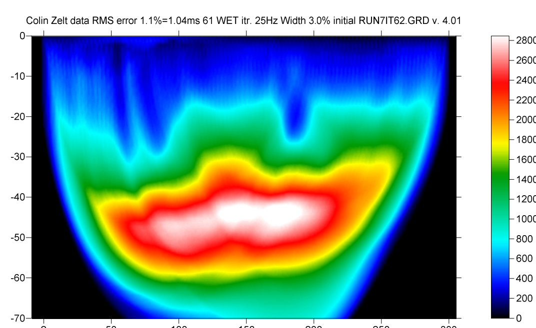

Fig. 34 : WET wavepath coverage plot for Fig. 32. Unit is wavepaths per pixel.

For Fig. 24 done in Dec 2020 with version 4.01 we used the same DeltatV and WET settings as for Fig. 21 (see Fig. 11) but we engaged our new ***Wavelength-Dependent Velocity Smoothing*** (WDVS; Zelt and Chen 2016) :

- forced cell size to 1.0m in *Header|Profile* see Fig. 22
- otherwise same DeltatV & Smooth invert & WET settings as in Fig. 11 / as above
- did not skip every 2nd shot for WET
- engaged WDVS at 75Hz (Fig. 27)
- checked *Model|Fast WDVS Smoothing*

Note the sharper imaging of top-of-basement in Fig. 24 compared with Fig. 21. Also note the deeper imaging of the modeled fault zone (offset 200m).

Fig. 24 took ***22 minutes on Apple 2017 iMac with Intel Core i5*** @ 2.3GHz running Windows 7 64-bit Pro in Parallels Desktop 14 on MacOS High Sierra. Fig. 30 took 25 minutes. Fig. 32 took 55 minutes on this 2017 iMac. On ***Apple 2020 macMini with Intel Core i5*** @ 3.0GHz Fig. 30 took 15 minutes and Fig. 32 took 34 minutes, using 4 cores in Parallels Desktop 16.

Here are the SEIS32.* profile database files for Fig. 24. Here are the Surfer .GRD files & .PAR files & Surfer 11 .SRF for Fig. 24.

Here are the SEIS32.* profile database files for Fig. 30. Here are the Surfer .GRD files & .PAR files & Surfer 11 .SRF for Fig. 30.

Here are the SEIS32.* profile database files for Fig. 32. Here are the Surfer .GRD files & .PAR files & Surfer 11 .SRF for Fig. 32.

(Zelt and Chen 2016) show the effect of varying the WDVS frequency in

Zelt, C. A. and J. Chen, Frequency-dependent traveltime tomography for near-surface seismic refraction data, Geophys. J. Int., 207, 72-88, 2016

The optimum choice of the *WDVS frequency* is subjective. The lower the WDVS frequency the stronger the contrast of imaged velocity anomalies in overburden and the more shallow the top-of-basement is imaged. Same applies when decreasing WDVS parameters *Regard nth node* and *Angle increment*. Compare Fig. 28 to Fig. 32 with Fig. 24.

We give you meaningful parameters for WET inversion and WDVS Smoothing to explore the non-unique solution space of the misfit function (Schuster, 1993). It is your job to navigate this solution space using these parameters and appropriate heuristics to arrive at a satisfying solution showing small RMS error and good correlation with a priori knowledge from boreholes, outcrops, trenching and other geophysical methods such as resistivity etc.

See our updated help file for description of WDVS parameters in chapter ***Forward model traveltimes***. Press F1 function key in ***Model|WDVS Smoothing dialog*** (Fig. 27) to display popup help window for current control. Use TAB key to switch focus between controls.

Review more updated tutorials showing WDVS :

http://rayfract.com/tutorials/epikinv.pdf
http://rayfract.com/tutorials/camp1.pdf
http://rayfract.com/tutorials/11REFR.pdf
http://rayfract.com/tutorials/jenny13.pdf
http://rayfract.com/tutorials/NORCAL14.pdf
http://rayfract.com/tutorials/Aaknes-1.pdf

For our latest release notes see

http://rayfract.com/help/release_notes.pdf

Conclusions

We show that engaging WDVS for WET inversion can improve the resolution in resulting tomograms. We show this with synthetic data where the top of the basement is imaged more sharply with stronger velocity contrast between overburden and basement. The lower the WDVS frequency the stronger the velocity contrast at top-of-basement and the more shallow the top-of-basement is imaged. WDVS will not help if reciprocal traveltime picking errors are too large or with errors in specification of used recording geometry. Also WDVS works best with a shot spacing of three receiver spacings or closer.

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