Multiscale Conjugate Gradient
SAGEEP11 with Rayfract® 3.34

Fig. 1: Basement step with fault model. See Zelt et al., 2013: blind test of refraction methods.

Fig. 2: 1D-Gradient starting model: laterally average $\Delta t/V$ inversion. RMS error 4.9%=7.80 ms.

Fig. 3: 4th WET run. Starting model is 3rd run output. Wavepath width 10%. RMS error 0.8%=1.20 ms.

Fig. 4: WET wavepath coverage plot for Fig. 3. Unit is wavepaths per pixel. Wavepath width 10%@25 Hz.

Fig. 5: 5th WET run. Starting model is 5th run output. Wavepath width 5%. RMS error 0.7%=1.08 ms.

Fig. 6: 6th WET run. Starting model is 6th run output. Wavepath width 3%. RMS error 0.6%=1.02 ms.

Fig. 7: WET wavepath coverage plot for Fig. 6. Unit is wavepaths per pixel. Wavepath width 3%@25 Hz.

Fig. 8: True times (colored solid curves forward modeled for Fig. 1 with added Gaussian noise) and inverted times (dashed blue curves for Fig. 6).
Fig. 9: Smooth inversion Settings

Fig. 10: WET Tomo | WET tomography Settings

Fig. 11: WET Tomo | Interactive WET tomography… (left), Edit velocity smoothing (right)
To obtain Fig. 2 to Fig. 8 proceed as following:

- Download 334CGWETGrad_smooth30_seis32.zip archive and unzip into directory C:\RAY32\SAGEEP11
- Select File|Open Profile... & database schema C:\RAY32\SAGEEP11\SEIS32.DBD. Click Open button.
- Check Smooth invert|Smooth inversion Settings|Extra-large cell size
- Setup other settings in Smooth invert|Smooth inversion Settings as in Fig. 9
- Uncheck WET Tomo|WET tomography Settings|Disable wavepath scaling for short profile
- Uncheck WET Tomo|WET tomography Settings|Scale wavepath width
- Check WET Tomo|WET tomography Settings|Edit maximum valid WET velocity
- Check WET Tomo|WET tomography Settings|Store modeled picks after last iteration only
- Setup other settings in WET Tomo|WET tomography Settings as in Fig. 10
- Select Smooth invert|WET with 1D-gradient initial model and confirm prompts to obtain 1D-gradient starting model (Fig. 2) and default Smooth inversion output after 20 WET iterations
- Select WET Tomo|Interactive WET tomography...
- Click Select button and select starting model C:\RAY32\SAGEEP11\GRADTOMO\GRADIENT.GRD
- Check box Skip every 2nd shot for forward modeling and click button Accept parameters
- Set edit field Wavepath frequency to 25Hz and set Max. velocity to 3,500m/s
- Click radio button Conjugate Gradient and uncheck box Steepest Descent step
- Edit other fields in WET main dialog as in Fig. 11 (left)
- Click button Edit velocity smoothing and radio button Manual specification of smoothing filter
- Set Half smoothing filter width to 3 columns and Half smoothing filter height to 2 grid rows
- Uncheck box Automatically adapt shape of rectangular filter matrix
- Set Maximum velocity update to 15% and Damping to 0.9
- Set Smooth nth iteration : n = 30 and click radio button Uniform
- Set Uniform Central row weight to 100 and uncheck box Smooth velocity update
- Edit other fields in Velocity Smoothing Parameters dialog as in Fig. 11 (right)
- Click button Accept parameters
- Click Iterate button in WET main dialog and click Reset button in Edit WET runs dialog
- Check box WET runs active and uncheck box Blank after last run
- Edit other fields in Edit WET runs dialog as in Fig. 12 and click OK button
- Click Edit grid file generation button and set Store each nth iteration only : n = 50.
- Click Accept parameters button and Start tomography processing button
- Confirm prompts to obtain output as in Fig. 3 to Fig. 7
- Select Refractor|Shot breaks to obtain Fig. 8
- Uncheck Mapping|Display raytraced traveltimes
With these velocity smoothing settings (Fig. 11) we obtain robust convergence of multirun WET inversion with RMS error quasi-monotonically decreasing from Fig. 2 (4.9%=7.80ms) through Fig. 6 (0.6%=1.02ms). In particular we specify:

- **Wavepath frequency of 25Hz** instead of default 50Hz. This makes multirun WET more robust.
- **Max. velocity = 3,500m/s**. This prevents unrealistic oscillation of Conjugate Gradient WET modeled velocity in basement.
- **Maximum velocity update = 15%** instead of default 25%
- **Damping = 0.9** equals default 0.9 for Conjugate Gradient method
- **Smooth nth iteration : n = 30** instead of default 1
- **Uniform central row weight = 100** instead of default value 1. This increases the vertical resolution in special case of quasi-horizontal layering in subsurface with Uniform Smoothing filter weighting.
- **Manual Smoothing filter specification** with Half-width = 3 columns and Half-height = 2 rows
- **No smoothing of velocity update** before applying the update to the current velocity tomogram

Fig. 3 (wavepath width 10%) is a low-frequency approximation of the true model (Fig. 1) and shows long-wavelength features of the true model. Fig. 6 (wavepath width 3%) is the final high-frequency interpretation of the traveltime data and shows more detail (short wavelength scale). By using above maximum velocity update of 15% and damping of 0.9 we keep the long wavelength features from Fig. 3 and add shorter wavelength resolution through all WET runs up to Fig. 6. Increasing the damping to 0.92 or even 0.95 better suppresses inversion artefacts at bottom of the dipping fault zone.

When using the Conjugate Gradient search method try enabling WET Tomo|WET tomography Settings|Blank|Blank low coverage after last iteration. This will blank out low-velocity edges at the bottom of the tomogram. These can occur because the Conjugate Gradient method combines previous velocity updates from all previous WET iterations (for current run) with the Steepest Descent direction for current iteration to determine the update for the current iteration (in new search direction).

To understand the Conjugate Gradient theory regard the velocity update grid as a vector with as many dimensions as there are cells in the tomogram grid i.e. vector dimensions = column count times row count of grid.

As stated in Zelt et al. 2013 “Uncorrelated Gaussianly-distributed noise with a mean of zero and a standard deviation of 1 ms was added to the synthetic data”. This added Gaussian noise prevents our WET inversion from reaching an even better resolution in the final tomogram Fig. 6.

Runtime for above 8 WET runs 62 WET iterations each = 496 iterations was about 6 minutes on an Apple iMac late 2015 with 2.8 GHz Intel Core i5 processor running Parallels Desktop 11 and Windows 7 Pro. Grid size is 80 rows x 200 columns. X spacing & Y spacing are 1.58m. One grid cell is 1.58m square. This fast runtime enables quasi-interactive variation of WET smoothing parameters (Fig. 11).

Try to speedup multirun Conjugate Gradient (CG) WET inversion by decreasing CG iterations from default 15 to 5 or 10 (Fig. 11 left) and/or decreasing Line Search iters. from default 3 to 2. CG iterations is the number of outer loop iterations, Line Search iters. is the number of inner loop iterations, $j_{max}$ in the algorithm shown on page 53 of Shewchuk, 1994. One inner iteration is done with one WET iteration. Our Tolerance parameter corresponds to Shewchuk epsilon $\epsilon$ for outer loop termination. Our Line Search tol. parameter corresponds to Shewchuk epsilon $\epsilon$ used for inner loop termination. Our Initial step parameter corresponds to Shewchuk sigma $\sigma$ used during first Secant method iteration in Equation (58) on page 46 of Shewchuk, 1994.

We have shown that using our default 1D-gradient starting model (Fig. 2) for multiscale tomography gives you a good vertical resolution (Fig. 6) comparable to the true model (Fig. 1). Sheehan et al. 2005 evaluate our 1D-Gradient starting model with Synthetic data generated for known subsurface models. Also our tutorial http://rayfract.com/tutorials/step.pdf written in 2013 using version 3.25 of our software shows that using our default 1D-Gradient starting model with Smooth inversion and 20 or 100 WET iterations gives a good vertical resolution.
Below we show using alternative WET parameters. For Fig. 13 (tutorial sageep11_smooth50.pdf) we used Steepest Descent method instead of Conjugate Gradient method, Damping 0.0 instead of 0.9, Smooth nth iteration : n = 50 and above 1D-gradient starting model (Fig. 2). Fig. 14 was obtained with same settings as for Fig. 13 but completely disabling WET smoothing by setting Smooth nth iteration : n = 100 and unchecking Smooth last iteration. This will normally not work for field recorded data due to measurement errors and too wide shot spacing.

Fig. 13 : 8th WET run. Starting model is 7th run output. Wavepath width 3%. RMS error 0.7% = 1.07ms. 1D-gradient starting model. Steepest Descent method. 50 WET iterations per run. Smooth nth iteration : n = 50. Smooth last iteration unchecked. Damping 0.0.

Fig. 14 : same as Fig. 13 but Smooth nth iteration : n = 100, no smoothing. RMS error 0.7% = 1.05ms.

Fig. 15 : 8th WET run. Starting model is 7th run output. Wavepath width 3%. RMS error 0.6% = 1.02ms. 1D-gradient starting model. Conjugate Gradient method. Smooth nth iteration : n = 30. Smooth last iteration unchecked. Damping 0.9. See Fig. 6.

For field-recorded data first run our Smooth inversion with Smooth invert|WET with 1D-gradient initial model, with default 20 and interactive 50 or 100 iterations. Next you can optionally decrease the smoothing and try Conjugate Gradient method instead of default Steepest Descent method, see above. Try Minimal smoothing, Smooth nth iteration : n = 4 or 10 and disabling Smooth velocity update (Fig. 11).

Quoted from our SAGEEP 2014 paper: “Besides showing the resolution limit of one wavelength (Watanabe et al., 1999), WET blurring also can show the uncertainty caused by bad picks, recording geometry errors, uncorrected trigger delays and out-of-plane refractions. The lower the signal-to-noise ratio, the wider the wavepaths should remain. Ray-based tomography methods tend to overfit the data and often generate artefacts (Zelt et al., 2013).”

Alternatively to the 1D-gradient starting model you can use a layered refraction starting model with our WET inversion. See tutorial norcal14.pdf.