

The DeltatV 1D method for seismic refraction inversion: Theory

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Abstract

We present an alternative approach to the interpretation of seismic first breaks, as first described in German by Helmut Gebrände. The DeltatV method assumes that subsurface velocity varies smoothly in the vertical direction. Unlike many refraction analysis methods DeltatV does not require the interactive assignment of traveltimes to hypothetical and mathematically idealized refractors. Sorting traveltimes by common midpoint (CMP) instead of common shot averages out the effects of dipping layers on traveltimes. The travelttime field is smoothed naturally by stacking CMP-sorted traveltime curves over a few adjacent CMP's. Then each CMP curve is independently inverted with the 1D DeltatV method. It is based on "seismic stripping" of assumed incremental layers with constant vertical velocity gradients and positive or negative velocity steps at layer boundaries.

The constant velocity gradient assumption means that seismic rays follow circular arc segments inside each layer modeled. As a consequence, rays can be reconstructed and treated analytically. The method estimates the layer bottom velocity from traveltimes and then inverts for the layer top velocity by solving a system of two equations numerically. Seismic stripping is equivalent to physically lowering source and receiver for each such ray to the top of the next lower layer. Inversion of these reduced traveltimes and offsets then resolves the properties of this lower layer.

The lowered sources act as point sources, following Huygens principle. DeltatV models diffraction of seismic rays at the top of an inferred low velocity layer. The method automatically

detects systematic time delays on CMP curves and translates these delays into velocity inversions. Estimated velocities and layer thicknesses are corrected for inferred velocity inversions. The resulting 1D velocity vs. depth models below each CMP are gridded and imaged as a 1.5D velocity model. To obtain a 1D initial model without artefacts, the horizontally averaged velocity vs. depth model can be extended laterally along the profile. Forward modeling of traveltimes for such 1D models has shown a good overall fit with picked times. True 2D traveltime tomography based on such smooth 1D initial models has been shown to work well.

Keywords

Seismics, Inversion, Theory, Mathematical formulation

Introduction

As described by (Alekseev 1973), (Giese 1976, chapters 5.1 and 6.11) and (Gebrande 1985, chapter 3.4.5.14), the crustal velocity varies in the vertical direction with two basic components: a systematic and continuous increase of velocity with depth, overlain by medium and small-scale random velocity oscillations. See (Gebrande 1985, Figure 3.93), as reproduced in Figure 1. Based on our experience with shot-sorted and CMP-sorted travelttime curves obtained from many field surveys, we assume that this description is valid for near surface velocity variation as well. To realistically image such random velocity changes in the vertical direction with constant velocity gradient layers, it is necessary to discretize the physical velocity vs. depth function with a large number of incremental layers. Positive or negative velocity steps are allowed at layer interfaces.

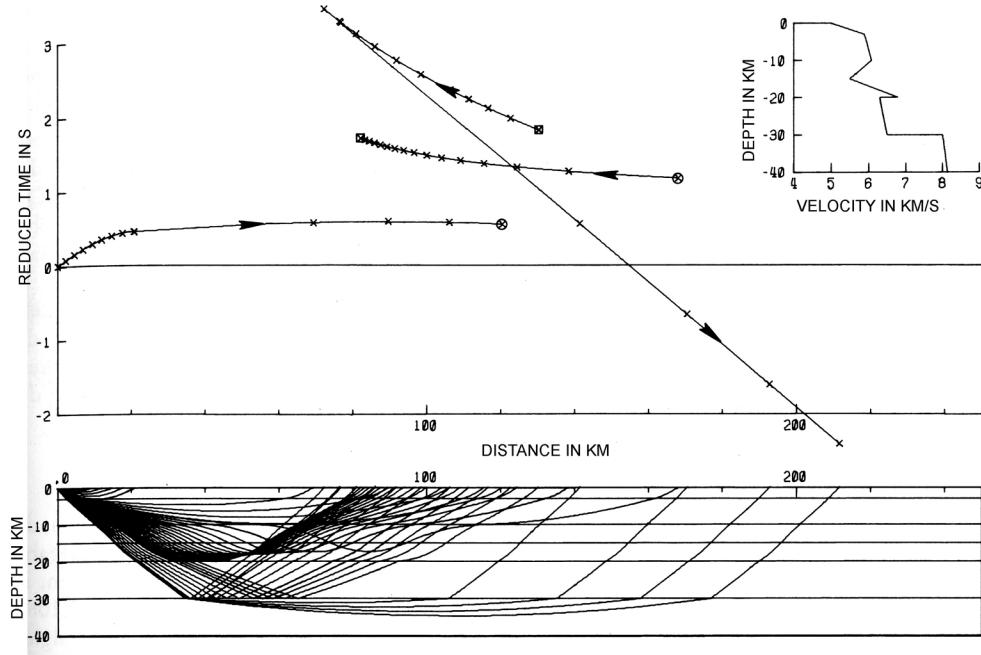


Figure 1 Rays and traveltime curves for a 1D continental crust model. The model is based on gradient layers. The 1D velocity vs. depth model function is shown at top right. Note the two velocity discontinuities. Reprinted, by permission of the author, from (Gebrande 1985, Figure 3.93).

One goal for the DeltatV method was to avoid the tedious, subjective and restrictive but conventionally required interpretation step of assigning common-shot-sorted traveltime curve segments to mathematically idealized “refractors”. These refractors are assumed to have no internal vertical velocity gradient. Velocity is allowed to change horizontally only, inside such a conventional refractor (Jones and Jovanovich 1985; Palmer 1980). As mentioned by MacPhail (1967), the assumption that the “velocity stratification” could be inferred unambiguously from common-shot-sorted traveltime curves is not very realistic. As soon as faults, local velocity anomalies, pinchouts, outcrops and vertical velocity gradients within layers are encountered, the assignment of first breaks to laterally continuous idealized refractors becomes almost impossible.

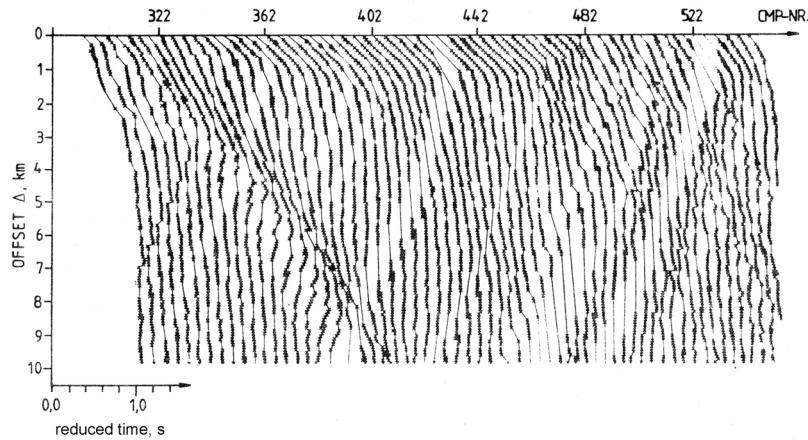
As described by MacPhail (1967) and Gebrande (1985, 1986), a logical solution to this problem is to sort first break traveltimes by common midpoint (CMP). First breaks assigned to the same CMP are sorted by absolute offset (distance between trace specific source and receiver). A CMP specific traveltime curve can be obtained by plotting traveltime t vs. absolute offset Δ . Effects of constant dips of individual layers on instantaneous velocity (CMP traveltime curve slope), are averaged out and removed to a large degree, as in seismic reflection processing (Diebold and Stoffa 1981). Each CMP curve is then inverted individually and iteratively with the DeltatV method (Gebrande 1985, 1986), to obtain a smooth 1D velocity vs. depth model below each CMP.

Another important motivation for the DeltatV method as stated by Gebrande (1986) was to obtain a good initial model for true 2D traveltime tomography.

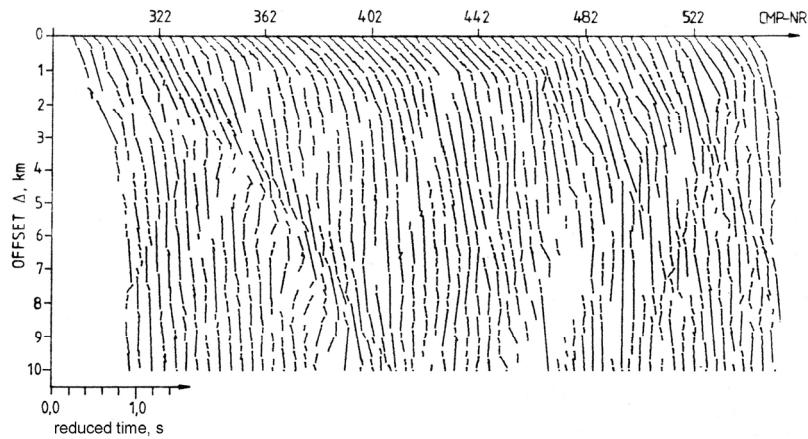
CMP travelttime field display

Averaging of first breaks recorded at common absolute offsets over a few adjacent CMP's reduces the effects of picking and geometry errors and smoothes the travelttime curve at the same time. Due to this travelttime field smoothing, low coverage refraction profiles (e.g., 7 shots into 24 receivers) can be processed as described below with reasonable success. However, the method works best for high coverage surveys.

To obtain a good first estimate of the 2D subsurface and the "instantaneous" velocity distribution, even before inversion of the travelttime data, CMP-sorted travelttime curves can be combined into one diagram, as shown in Figure 2. The horizontal axis is the CMP station number. The vertical axis (increasing downwards) is the absolute offset Δ . Each smoothed CMP travelttime curve is then plotted in this diagram with the travelttime curve origin attached to the CMP on the top horizontal axis (at $\Delta 0$).



Top : raw data, bin width of 8 adjacent CMP gathers.



Bottom : optimized piecewise linearization of CMP traveltime curves. The standard deviation between raw data and linearized times has been minimized.

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Figure 2 Plot of first breaks along profile DEKORP 2 in reduced form ($v_{\text{red}} = 6.0 \text{ km/s}$).

Δ - t tuples are plotted in a local coordinate system with the origin at the curve specific CMP. The vertical axis of this local coordinate system is the Δ axis, the same axis as used for the whole diagram. The horizontal local axis is the time axis, with time t increasing to the right (Figure 2). An increasing slope of a travelttime curve segment corresponds to an increasing apparent or instantaneous subsurface velocity at the corresponding Δ . Increasing Δ corresponds to increasing depth penetration of rays. By plotting distance Δ vs. reduced travelttime $t-\Delta/v_{\text{red}}$,

inflections can be exaggerated and local instantaneous velocity variations can be enhanced, as with conventional refraction analysis.

The next processing step is the individual inversion of each smoothed $\Delta-t$ curve with the 1D DeltatV method, as described below. The inversion assumes that there are many horizontal layers (nonsloping layer case) with constant internal vertical velocity gradients and no lateral velocity variation below each CMP. This assumption is not too restrictive because constant layer dip effects on instantaneous CMP velocity are averaged out as discussed earlier.

DeltatV method theory

As shown by Gibson et al. (1979), the assumption that the vertical velocity gradient within each layer is constant makes it possible to model layer internal total refraction of rays analytically with simple parametric traveltimes equations. Since the velocity gradient is constant seismic rays follow circular arc segments inside each layer modeled (Gebrande 1985, 1986). For a horizontal layer of thickness h , with a ray grazing the layer's bottom and corresponding shot point and receiver located at the top of the layer, the following relations hold:

$$\Delta = \frac{2\sqrt{1-p^2b^2}}{pb} \gamma \quad (1)$$

$$t = \frac{2}{\gamma} \cdot \log\left(1 + \frac{\sqrt{1-p^2b^2}}{pb}\right) \quad (2)$$

$$\beta = b + \gamma h \quad (3)$$

$$p = \frac{1}{\beta} \quad (4)$$

where

Δ (referred to as "x" by Gibson et al.) is the absolute offset between shot point and receiver,
 t is the traveltime along the ray from shot point to receiver,
 p is the ray parameter (see Gibson 1979, page 186),
 γ is the layer internal velocity gradient,
 b is the velocity at the top of the layer (velocity at $z = 0$) and
 β is the velocity at the bottom of the layer (velocity at $z = h$).

Gebrande uses these symbols: a for γ , v_0 for b and V for β .

Under the simplifying assumption that there are no strong vertical velocity discontinuities, β can be determined directly from the CMP-sorted traveltime curve. This velocity estimation is done by linear regression over a few $\Delta-t$ tuples. These tuples are contained in a Δ interval centered at the assumed Δ value for the ray grazing the bottom of the layer currently being processed, as shown in Figure 3.

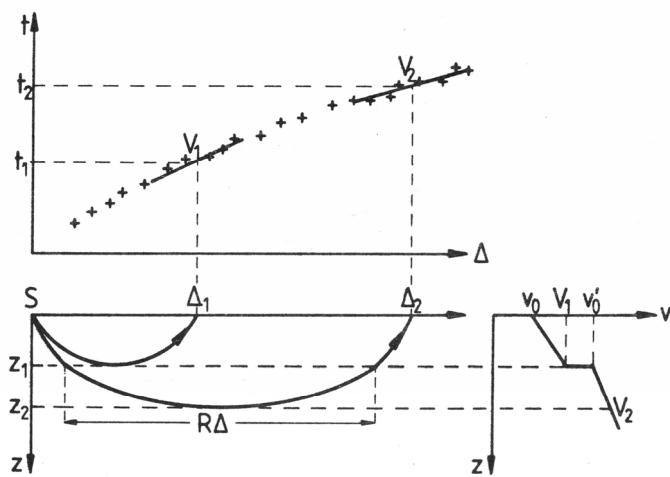


Figure 3 X-T-V inversion is based on iterative stripping of gradient layers.
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Since b , the desired result, is defined by above equations (1) and (2) implicitly (it is not possible to resolve for b explicitly), b must be determined numerically. This is done with the standard Newton-Raphson root finding method for an implicit function $f(b)$. The method is made fail-safe by a hybrid algorithm that takes a bisection step whenever Newton-Raphson does not work. See (Press et al. 1986), paragraph 9.4.

In the following, two well known mathematical identities for the hyperbolic function arc cosh are used :

$$\text{arccosh}(x) = \log(x + \sqrt{x^2 - 1}) \quad (5)$$

$$\frac{d}{dy} \text{arccosh}(y) = \frac{1}{\sqrt{y^2 - 1}} \quad (6)$$

Using relationship (4) for p , equation (1) from above can be reformulated (Gebrande 1986) as

$$\Delta(\beta) = \frac{2}{\gamma} \sqrt{\beta^2 - b^2} \quad (7)$$

Also, when substituting $\frac{1}{pb}$ for x in (5), equation (2) from above can be rewritten (again using (4)) as

$$t(\beta) = \frac{2}{\gamma} \text{arcosh}\left(\frac{\beta}{b}\right) \quad (8)$$

When resolving (7) for γ and inserting the resulting expression into (8), the implicit function $f(b)$ expected by Newton-Raphson becomes

$$f(b) = \frac{t\sqrt{\beta^2 - b^2}}{\Delta} - \operatorname{arccosh}\left(\frac{\beta}{b}\right) = 0 \quad (9)$$

Using the chain rule for derivation of composed functions and when substituting β / b for y in (6), derivation of $f(b)$ after b yields $f'(b)$, as required by Newton-Raphson :

$$f'(b) = \frac{\beta/b^2 - t/\Delta}{\sqrt{(\beta/b)^2 - 1}} \quad (10)$$

The largest possible value for b (in the special case of a horizontal flat ray, $b = \beta$ and $\gamma = 0$) is

$$b_{\max} = \sqrt{\beta \frac{\Delta}{t}} \quad (11)$$

So one DeltatV iteration (for determining one layer) is obtained as follows: first, a plausible candidate $\Delta-t$ tuple is determined from the $\Delta-t$ traveltime curve. This tuple represents the ray grazing the bottom of that layer. Then, the layer bottom velocity β is estimated at that Δ with linear regression, as described above. Candidate rays for which β exceeds a user specified maximum acceptable velocity value are skipped. Now Newton-Raphson with bisection is carried out using (9) and (10) for the b interval bracketed by the minimum value 0 and maximum value b_{\max} (11). Candidate rays and $\Delta-t$ tuples for which b_{\max} (11) equals or exceeds β are skipped.

Once an estimate for the layer top velocity b has been obtained in this way, the CMP and layer specific constant vertical velocity gradient can be obtained by resolving (7) for γ :

$$\gamma = \frac{2}{\Delta} \sqrt{\beta^2 - b^2} \quad (12)$$

And finally the desired output, layer thickness, is derived from (3) as

$$h = \frac{\beta - b}{\gamma} \quad (13)$$

Once the topmost layer has been determined in this way, all rays with a Δ larger than the Δ assumed for the topmost layer bottom are corrected for the effects of this topmost layer as shown by Gibson et al. (1979). Absolute offset Δ and travelttime t are reduced according to Gibson equations (7a) and (7b), with a k value of 2 ("overburden layer stripping", 2-layer case). This reduction corresponds to physically lowering the source and receiver for each such ray to the top of the next lower layer. Now the lowered sources act as point sources according to Huygens principle.

Parameters for the next lower layer (now the topmost layer itself) are determined by inversion during the next DeltatV iteration by first estimating β' at the newly assumed (reduced) Δ' for this layer's bottom, repeating the Newton-Raphson procedure, and then solving for γ and h with equations (12) and (13). The resulting b' for the top of this layer can be larger than, equal to or smaller than the estimated β for the bottom of the overlying (previously determined) layer. Therefore, diffraction of seismic rays at the top of a low velocity layer (LVL) is modeled correctly. Velocity inversions can be recognized, provided that the average receiver separation (receiver

spacing) and the shot spacing are narrow enough and the LVL is thick enough (Rohdewald 2010c).

This process is repeated iteratively until there are no more Δ -t tuples (representing rays) with reduced Δ values exceeding the Δ value assumed during inversion of the last layer.

Winkelmann (1998) describes an extended DeltatV method (extended XTV inversion), which detects and models both constant velocity gradient layers and constant velocity layers.

DeltatV implementation

As described above, Gebrande proposes to image random velocity changes in the vertical direction with a large number of incremental constant gradient layers, with velocity steps at layer interfaces. But for shallow engineering type surveys with about 10 shots into 24 receivers, the number of distinct Δ -t tuples per CMP travelttime curve will be limited. The Δ spacing of these tuples will be relatively wide, even after smoothing and complementing the curve with Δ -t tuples from a few neighboring CMP gathers. The only solution to this dilemma is to smooth laterally over a relatively wide CMP interval, e.g., over 20 to 30 CMP's. For long profiles, this can be increased to 100 or 200 CMP's (Rohdewald 2010c, Rohdewald 2011b).

We recommend a three-step solution: First, weight individual first breaks, e.g., with the reciprocal of the square root of the CMP offset. Second, determine the optimum stack width and trace weight interactively in the CMP travelttime field display (see above). Third and finally, estimate the layer bottom velocity β by linear regression over just 3 or 5 Δ -t tuples belonging to one smoothed CMP travelttime curve. Although in theory not necessary, it is beneficial to optionally add a tuple with Δ -t value (0, 0) to such a curve, located at the (reduced) Δ -t origin.

This prevents too high estimates of apparent velocity β , and makes the inversion more robust, in case of noisy traces and bad picks. This option can be turned off, in our DeltatV implementation.

The first DeltatV iteration will then be done for the smallest Δ , referred to as Δ_1 in the following discussion. The layer bottom velocity β_1 for that Δ_1 will be estimated by linear regression over the three tuples $(0, 0)$, (Δ_1, t_1) and (Δ_2, t_2) . Once the parameters of the topmost layer have been resolved as described above, all Δ -t tuples with $\Delta > \Delta_1$ will be reduced according to Gibson (7a) and (7b). Now the second DeltatV iteration will be done at Δ_2' , the reduced Δ_2 . β_2' will be obtained by linear regression over tuples $(0, 0)$, (Δ_2', t_2') and (Δ_3', t_3') etc.

The layer bottom velocity β typically increases most rapidly for the uppermost few layers. Therefore, linear regression over about 3 tuples is recommended to obtain adequate resolution of weathering velocity vs. depth. The relative velocity increase per layer (compared to directly overlying layer) will decrease with increasing layer number and ray penetration depth. At the same time, the relative error in reduced Δ -t tuples will increase with depth and with each DeltatV iteration. Therefore we propose to increase the number of Δ -t tuples required for estimation of β to about 5 for these deeper layers.

In situations where the topography undulates strongly along the 2D profile, it is necessary to compute and apply static corrections to traces before DeltatV inversion. CMP gather specific static corrections may be computed for a dipping plane, approximating source and receiver elevations of all traces mapped to one stacked CMP curve. Alternatively, surface consistent static corrections may be determined for a smoothed topography determined by applying a running average filter to all profile station elevations.

Imaging and processing of DeltatV output

For gridding and 2D contouring of the 1.5D DeltatV tomogram, a unique velocity value needs to be determined for each interface below each CMP (Gebrande 1986, Figure 7). b for the current layer is reconciled with β for the overlying layer for all layer interfaces imaged by taking the (minimum or) average of these two values and regarding it as the correct velocity value. Note the ambiguity and non-uniqueness, as illustrated in Figure 4.

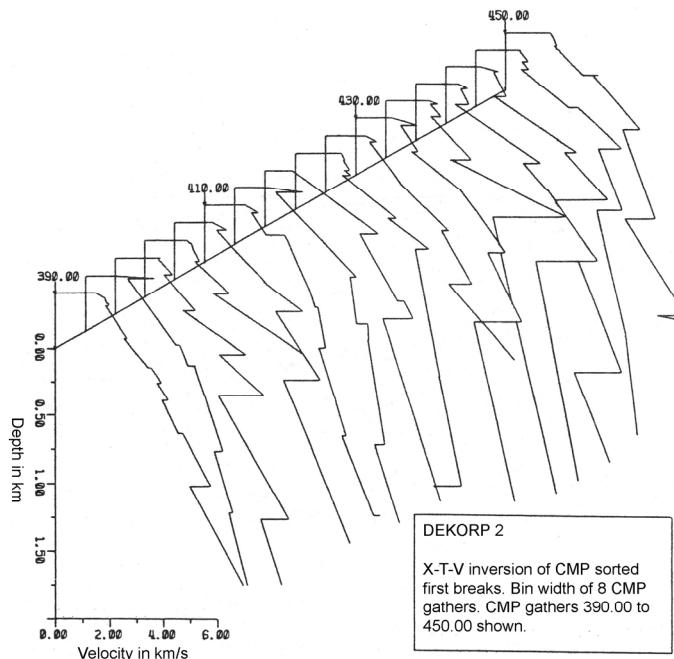


Figure 4 X-T-V inversion of CMP-sorted traveltimes curves shown in Figure 1. The resulting 1D $v(z)$ models have been combined perspectively into a 2D model. Not every detail of the velocity distribution is significant, since no later events (reflections) have been used.

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author, from (Gebrande 1986, Figure 7).

Alternatively, the horizontally averaged 1D velocity vs. depth model as obtained with DeltatV can be extended laterally along the profile. This will filter out systematic DeltatV artefacts due to the ray geometry at an early stage, in case of strong lateral variation of subsurface velocity. The resulting gradient model can then be used as an initial model for, e.g., Wavepath Eikonal

Traveltime (WET) true 2D inversion (Schuster 1993). The initial fit between picked and forward modeled times (Lecomte et al. 2000) determined for the 2D DeltatV model grid is quite close in most geological settings already. As shown by (Sheehan 2005) and (Jansen 2010) in his Appendix C, smoothed DeltatV 1D initial model refined by WET gives models which match the true model well. In our experience, WET redistributes velocity laterally only. The average background vertical velocity profile does not change. For our own synthetic modeling, see (Rohdewald 2010a), (Rohdewald 2010b). (Rohdewald 2011a) shows modeling of a thrust fault.

The DeltatV method allows the consideration of first breaks picked at far offset shot traces, and thus can estimate velocities beyond the ends of the geophone spread. But true 2D tomography such as WET inversion can only consider shots positioned between or directly adjacent to profile receivers.

For surveys with homogeneous overburden, including marine surveys recorded with streamers, DeltatV can work well, even without running WET inversion. See (Rohdewald 2010c, Rohdewald 2011a, Rohdewald 2011b). The longer the seismic line, recorded with roll-along technique customary in reflection seismics, and the denser the receiver and shot spacing, the better DeltatV will work.

Discussion

Benefits of the DeltatV Method

DeltatV does not require the user to subjectively assign traveltimes to hypothetical refractors. The method naturally images most features of 2D subsurface velocity distributions, including local anomalies, faults, pinchouts, outcrops etc. With increasing burial depth, the degree of weathering typically decreases (Murck 2001, chapter 6) and overburden pressure and resulting compaction increases. The resulting systematic velocity increase with depth is modeled realistically with the DeltatV assumption of a piecewise constant velocity gradient. Estimated

velocities and layer thicknesses are automatically corrected for inferred velocity inversions. The method can be implemented run-time efficiently and thus allows for the interactive variation of parameters. While the tau-p method (Diebold and Stoffa 1981) assumes that velocity increases monotonically with depth, DeltatV allows for, and sometimes infers and models velocity inversions. DeltatV automatically detects systematic time delays on CMP curves and models these as velocity inversions. (McMechan 1979a; McMechan 1979b) describes interpretation of low-velocity zones with Wiechert-Herglotz integrals. DeltatV does not use the intercept time tau. Avoiding this extrapolation of the current refractor segment to zero-offset (Figure 3) can make the inversion more robust.

Comparison with conventional methods

Conventional seismic refraction methods such as the Plus-Minus method (Hagedoorn 1959) and Generalized Reciprocal Method (GRM) (Palmer 1980) project average refractor-internal velocities to the top of the refractor. The DeltatV method decompresses this velocity information and naturally images ray bottom velocities at their appropriate depth.

In general, these conventional methods resolve local anomalies of common-shot-sorted traveltimes with lateral velocity variation along a quasi-1D interface represented by the top of the idealized refractor. Our 1.5D method allows the modeled velocity to vary quasi-continuously, both in vertical and lateral direction, to explain CMP traveltimes field anomalies and inflections. For each ray the instantaneous velocity or ray parameter p (Gibson et al. 1979) is mapped automatically to the ray's maximum depth (Gebrande 1986, Figure 4 and Figure 7).

This maximum depth is the depth at which the ray “turns” back to the surface and is critically refracted. The offset is the ray's absolute source-receiver distance, Δ , and varies quasi-continuously between 0 and the maximum offset recorded below one CMP. Maximum ray depth increases monotonically with absolute offset and is allowed to increase in laterally partially constrained depth steps, for increasing incremental layer number below each CMP. The wider

the CMP stack width and the more weight is given to CMP's offset from the central CMP, the more laterally constrained DeltatV inversion will be. The 1.5D, sometimes approaching 2D velocity variation is modeled by gridding of these 1D depth-velocity profiles. Lateral velocity resolution below one CMP and at a specific depth is related to the absolute source-receiver offset Δ of the ray which is critically refracted at that approximate depth. Also, the shape of the basement below this CMP influences lateral resolution (Rohdewald, 2010a).

With the conventional methods, velocity and depth modeled below one station can only assume one discrete value pair per refractor modeled. The maximum allowed number of such mathematically idealized refractors (with no layer-internal vertical velocity gradient) is small (about 3 to 5). Also, the conventional restricting assumption is made that these refractors are laterally continuous and exist over the whole length of the profile. The DeltatV method does not assume laterally continuous layers. The maximum number of layers modeled below one CMP is large (about 10 to 20, depending on shot point spacing and receiver spacing and on the largest offset recorded). So our method relaxes the conventional discrete-layer velocity model into a quasi-continuous 1.5D velocity model.

Velocity is conventionally assumed to increase with depth in only a few large, i.e., decidedly discrete, positive steps. DeltatV allows for quasi-continuous velocity variation with depth and velocity inversions (velocity decreasing with depth). The vertical resolution of velocity depends on the receiver spacing and the shot spacing. The closer the spacings used in the field and the longer the overlapping receiver spreads, the higher the resolution will be.

Conclusions

With manual processing of seismic refraction data, it is appropriate to model the subsurface velocity distribution with only a few constant vertical velocity refractors. With powerful desktop and portable computers, however, we find that it is appropriate to relax conventional

discrete-layer velocity modeling into quasi-continuous modeling of subsurface velocity. As a consequence faults and other local velocity and related structural anomalies can now be imaged more reliably. True 2D travelttime tomography based on initial models obtained with the DeltatV method has been shown to work well.

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